Fixed points of quantum gravity in extra dimensions

Peter Fischer[a] and Daniel F. Litim[b,c]

[a] *Institut für Theoretische Physik E, RWTH Aachen, D - 52056 Aachen*

[b] *School of Physics and Astronomy, U Southampton, Highfield, SO17 1BJ, U.K.*

[c] *Physics Department, CERN, Theory Division, CH – 1211 Geneva 23.*

Abstract

We study quantum gravity in more than four dimensions with renormalisation group methods. We find a non-trivial ultraviolet fixed point in the Einstein-Hilbert action. The fixed point connects with the perturbative infrared domain through finite renormalisation group trajectories. We show that our results for fixed points and related scaling exponents are stable. If this picture persists at higher order, quantum gravity in the metric field is asymptotically safe. We discuss signatures of the gravitational fixed point in models with low scale quantum gravity and compact extra dimensions.

The physics of gravitational interactions in more than four space-time dimensions has received considerable interest in recent years. The possibility that the fundamental Planck mass – within a higher dimensional setting – may be as low as the electroweak scale [1, 2, 3] has stimulated extensive model building and numerous investigations aiming at signatures of extra spatial dimensions ranging from particle collider experiments to cosmological and astrophysical settings. Central to these scenarios is that gravity lives in higher dimensions, while standard model particles are often confined to the four dimensional brane (although the latter is not crucial in what follows). In part, these models are motivated by string theory, where additional spatial dimensions arise naturally [4]. Then string theory would, at least in principle, provide for a short distance definition of these theories which presently have to be considered as effective rather than fundamental ones. In the absence of an explicit ultraviolet completion, gravitational interactions at high energies including low scale gravity can be studied with effective field theory or semi-classical methods, as long as quantum gravitational effects are absent, or suppressed by some ultraviolet cutoff of the order of the fundamental Planck mass, *e.g.* [5].

One may then wonder whether a quantum theory of gravity in the metric degrees of freedom can exist in four and more dimensions as a cutoff-independent, well-defined and non-trivial local theory down to arbitrarily small distances. It is generally believed that the above requirements imply the existence of a non-trivial ultraviolet fixed point under the renormalisation group, governing the short-distance physics. The corresponding fixed point action then provides a microscopic starting point to access low energy phenomena of quantum gravity. This ultraviolet completion does apply for quantum gravity in the vicinity of two dimensions, where an ultraviolet fixed point has been identified with ϵ-expansion techniques [6, 7, 8]. In the last couple of years, a lot of efforts have been put forward to access the four-dimensional case, and a number of independent studies have detected an ultraviolet fixed point using functional and renormalisation group methods in the continuum [9, 10, 11, 12, 13, 14, 15, 16, 17, 18] and Monte Carlo simulations on the lattice [19, 20].

Continuity in the dimension suggests that a non-trivial fixed point – if it exists in four dimensions and below – should persist at least in the vicinity and above four dimensions. Furthermore, the critical dimension of quantum gravity – the dimension where the gravitational coupling has vanishing canonical mass dimension – is two. For any dimension above the critical one, the mass dimension of the gravitational coupling is negative. Hence, from a renormalisation group point of view, four dimensions are not special. More generally, one expects that the local structure of quantum fluctuations, and hence local renormalisation group properties of quantum theories of gravity, are qualitatively similar for all dimensions above the critical one, modulo topological effects in specific dimensions.

In this Letter, we perform a fixed point search for quantum gravity in more than four dimensions [15] (see also [13]). An ultraviolet fixed point, if it exists, should already be visible in the purely gravitational sector, to which we confine ourselves. Matter degrees of freedom and gauge interactions can equally be taken into account. We employ a functional renormalisation group based on a cutoff effective action Γ_k for the metric field [9, 10, 11, 12, 13, 14, 15, 16, 17, 21], see [22] and [23] for reviews in scalar and gauge theories. In Wilson's approach, the functional Γ_k comprises momentum fluctuations down to the momentum scale k, interpolating between Γ_Λ at some reference scale $k = \Lambda$ and the full quantum effective action at $k \to 0$. The variation of the effective action with the cutoff scale ($t = \ln k$) is given by an exact functional flow

$$\partial_t \Gamma_k = \tfrac{1}{2} \text{Tr} \frac{1}{\Gamma_k^{(2)} + R_k} \partial_t R_k \,. \tag{1}$$

The trace is a sum over fields and a momentum integration, and R_k is a momentum cutoff for the propagating fields. The flow relates the change in Γ_k with a loop integral over the full cutoff propagator. By construction, the flow (1) is finite and, together with the boundary condition Γ_Λ, defines the theory. In renormalisable theories, the cutoff Λ can be removed, $\Lambda \to \infty$, and $\Gamma_\Lambda \to \Gamma_*$ remains well-defined for arbitrarily short distances. In

perturbatively renormalisable theories, Γ_* is given by the classical action, *e.g.* in QCD. In perturbatively non-renormalisable theories, proving the existence (or non-existence) of a short distance limit Γ_* is more difficult. In quantum gravity, the functional Γ_* should at least contain those diffeomorphism invariant operators which display relevant or marginal scaling in the vicinity of the fixed point. A fixed point action qualifies as a fundamental theory if it is connected with the correct long-distance behaviour by finite renormalisation group trajectories Γ_k.

The flow (1) is solved by truncating Γ_k to a finite set of operators, which can systematically be extended. Highest reliability and best convergence behaviour is achieved through an optimisation of the momentum cutoff [24, 25, 26, 27]. We employ the Einstein-Hilbert truncation where the effective action, apart from a classical gauge fixing and the ghost term, is given as

$$\Gamma_k = \frac{1}{16\pi G_k} \int d^D x \sqrt{g} \left[-R(g) + 2\bar{\lambda}_k \right] . \tag{2}$$

In (2), g denotes the determinant of the metric field $g_{\mu\nu}$, $R(g)$ the Ricci scalar, G the gravitational coupling constant, and $\bar{\lambda}$ the cosmological constant. In the domain of classical scaling G_k and $\bar{\lambda}_k$ are approximately constant, and (2) reduces to the conventional Einstein-Hilbert action in D euclidean dimensions. The dimensionless renormalised gravitational and cosmological constants are

$$\begin{aligned} g_k &= k^{D-2} \, G_k \; \equiv k^{D-2} \, Z_{N,k}^{-1} \, \bar{G} \\ \lambda_k &= \; k^{-2} \, \bar{\lambda}_k \; , \end{aligned} \tag{3}$$

where $\bar{G}$ and $\bar{\lambda}$ denote the couplings at some reference scale, and $Z_{N,k}$ the wave function renormalisation factor for the newtonian coupling. Their flows are given by

$$\begin{aligned} \partial_t g &\equiv \beta_g = \left[D - 2 + \eta_N \right] g \\ \partial_t \lambda &\equiv \beta_\lambda \end{aligned} \tag{4}$$

with $\eta_N(\lambda, g) = -\partial_t \ln Z_{N,k}$ the anomalous dimension of the graviton. Fixed points correspond to the simultaneous vanishing of (4). Explicit expressions for (4) and η_N follow from (1) by projecting onto the operators in (2), using background field methods. We employ a momentum cutoff with the tensor structure of [9] (Feynman gauge) and optimised scalar cutoffs (see below). For explicit analytical flow equations, see [15, 17]. The ghost wave function renormalisation is set to $Z_{C,k} = 1$. Diffeomorphism invariance can be controlled by modified Ward identities [9], similar to those employed for non-abelian gauge theories [28].

Two comments are in order. Firstly, the cosmological constant λ obeys $\lambda < \lambda_{\text{bound}}$, where $2\lambda_{\text{bound}} \equiv \min_{q^2/k^2}[(q^2 + R_k(q^2))/k^2]$ depends on the momentum cutoff $R_k(q^2)$ and $q^2 \geq 0$ denotes (minus) covariant momentum squared. Elsewise the flow (1), (4) could develop a pole at $\lambda = \lambda_{\text{bound}}$. The property $\lambda < \lambda_{\text{bound}}$ is realised in any theory where $\Gamma_k^{(2)}$ develops negative eigenmodes, and simply states that the inverse cutoff propagator $\Gamma_k^{(2)} + R_k$ stays
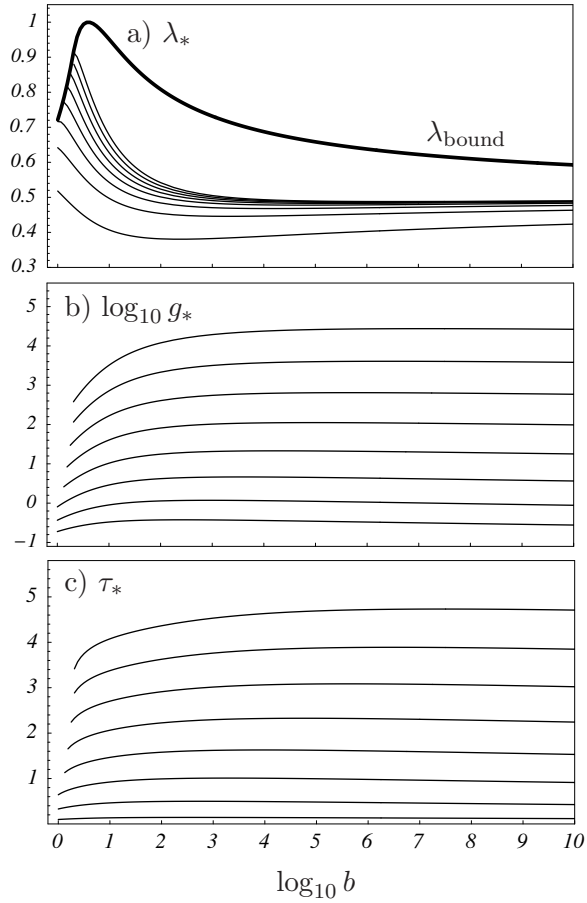
3

Figure 1: Fixed points in $D = 4 + n$ dimensions with $n = 0, ..., 7$ (thin lines from bottom to top) as a function of the cutoff parameter b for momentum cutoff r_{mexp}; a) the cosmological constant λ_* and λ_{bound} (thick line); b) the gravitational coupling g_*; c) the scaling variable $\tau_* = \lambda_* \, (g_*)^{2/(D-2)}$.

positive (semi-) definite [29]. Secondly, we detail the momentum cutoffs for the numerical analysis. We introduce $R_k(q^2) = q^2 \, r(y)$, where $y = q^2/k^2$. Within a few constraints regulators can be chosen freely [24]. We employ $r_{\mathrm{mexp}} = b/((b+1)^y - 1)$, $r_{\mathrm{exp}} = 1/(\exp cy^b - 1)$, $r_{\mathrm{mod}} = 1/(\exp[c(y + (b-1)y^b)/b] - 1)$, with $c = \ln 2$, and $r_{\mathrm{opt}} = b(1/y - 1)\theta(1 - y)$. These cutoffs include the sharp cutoff ($b \to \infty$) and asymptotically smooth Callan-Szymanzik type cutoffs $R_k \sim k^2$ as limiting cases, and b is chosen from $[b_{\mathrm{bound}}, \infty]$, where b_{bound} stems from $\lambda < \lambda_{\mathrm{bound}}$ [26]. The larger the parameter b, for each class, the 'sharper' the corresponding momentum cutoff.

Next we summarise our results for non-trivial ultraviolet fixed points $(g_*, \lambda_*) \neq (0, 0)$ of (4), the related universal scaling exponents, trajectories connecting the fixed point with the perturbative infrared domain, the graviton anomalous dimension, cutoff independence, and the stability of the underlying expansion. We restrict ourselves to $D = 4 + n$ dimensions,

τ_*								
r_{mexp}	0.132	0.461	0.933	1.502	2.142	2.834	3.568	4.336
r_{exp}	0.134	0.468	0.946	1.521	2.165	2.858	3.591	4.356
r_{mod}	0.135	0.469	0.946	1.521	2.162	2.853	3.585	4.348
r_{opt}	0.137	0.478	0.963	1.544	2.192	2.888	3.623	4.389
n	0	1	2	3	4	5	6	7

Table 1: Scaling variable τ_* in $D = 4 + n$ dimensions for various momentum cutoffs (see text).

with $n = 0, ..., 7$ (see Figs. 1 and 2, and Tab. 1 – 4).

Existence.— A real, non-trivial, ultraviolet fixed point exists for all dimensions considered, both for the cosmological constant and the gravitational coupling constant. Fig. 1 shows our results for λ_* and $\log_{10} g_*$ based on the momentum cutoff r_{mexp} with parameter b up to 10^{10}. For small b, their numerical values depend strongly on b, while for large b, they become independent thereof. Results similar to Fig. 1 are found for all momentum cutoffs indicated above [26].

Continuity.— The fixed points λ_* and g_*, as a function of the dimension, are continuously connected with their perturbatively known counterparts in two dimensions [10, 11, 15, 17].

Uniqueness.— This fixed point is unique in all dimensions considered.

Positivity of the gravitational coupling.— The gravitational coupling constant only takes positive values at the fixed point. Positivity is required at least in the deep infrared, where gravity is attractive and the renormalisation group running is dominated by classical scaling. Since the flow β_g in (4) is proportional to g itself, and the anomalous dimension stays finite for small g, it follows that renormalisation group trajectories cannot cross the line $g = 0$ for any finite scale k. Therefore the sign of g is fixed along any trajectory, and positivity in the infrared requires positivity already at an ultraviolet fixed point. At the critical dimension $D = 2$, the gravitational fixed point is degenerate with the gaussian one $(g_*, \lambda_*) = (0, 0)$, and, consequently, takes negative values below two dimensions.

Positivity of the cosmological constant.— At vanishing λ, β_λ is generically non-vanishing. Moreover, it depends on the running gravitational coupling. Along a trajectory, therefore, the cosmological constant can change sign by running through $\lambda = 0$. Then the sign of λ_* at an ultraviolet fixed point is not determined by its sign in the deep infrared. We find that the cosmological constant takes positive values at the fixed point, $\lambda_* > 0$, for all dimensions and cutoffs considered. In pure gravity, the fixed point λ_* takes negative values only in the vicinity of two dimensions. Once matter degrees of freedom are coupled to the theory, the sign of λ_* can change, *e.g.* in four dimensions [14]. We expect this pattern to persist also in the higher-dimensional case.

θ'								
r_{mexp}	1.51	2.80	4.58	6.71	9.14	11.9	14.9	18.2
r_{exp}	1.53	2.83	4.60	6.68	9.03	11.6	14.5	17.6
r_{mod}	1.51	2.77	4.50	6.54	8.86	11.4	14.2	17.3
r_{opt}	1.48	2.69	4.33	6.27	8.46	10.9	13.5	16.4
n	0	1	2	3	4	5	6	7

Table 2: Scaling exponent θ' in $D = 4 + n$ dimensions for various momentum cutoffs (see text).

Dimensional analysis.— In pure gravity (no cosmological constant term), only the sign of the gravitational coupling is well-defined, while its size can be rescaled to any value by a rescaling of the metric field $g_{\mu\nu} \rightarrow \ell\, g_{\mu\nu}$. In the presence of a cosmological constant, however, the relative strength of the Ricci invariant and the volume element can serve as a measure of the coupling strength. From dimensional analysis, we conclude that

$$\tau_k = \bar{\lambda}_k \, (G_k)^{\frac{2}{D-2}} \tag{5}$$

is dimensionless and invariant under rescalings of the metric field [30]. Then the on-shell effective action is a function of (5) only. In the fixed point regime, τ_k reduces to $\tau_* = \lambda_* (g_*)^{2/(D-2)}$. In Fig. 1c), we have displayed τ_* using the cutoff r_{mexp} for arbitrary b. In comparison with the fixed point values in Figs. 1a,b), τ_* only varies very mildly as a function of the cutoff parameter b, and significantly less than both g_* and λ_*. This shows that (5) qualifies as a universal variable in general dimensions. In Tab. 1, we have collected our results for (5) at the fixed point. For all dimensions shown, τ_* displays only a very mild dependence on the cutoff function.

Universality.— The fixed point values are non-universal. Universal characteristics of the fixed point are given by the eigenvalues $-\theta$ of the Jacobi matrix with elements $\partial_x \beta_y|_*$ and x, y given by λ or g, evaluated at the fixed point. The Jacobi matrix is real though not symmetric and admits real or complex conjugate eigenvalues. For all cases considered, we find complex eigenvalues $\theta \equiv \theta' \pm i\theta''$. In the Einstein-Hilbert truncation, the fixed point displays two ultraviolet attractive directions, reflected by $\theta' > 0$. Complex scaling exponents are due to competing interactions in the scaling of the volume invariant $\int \sqrt{g}$ and the Ricci invariant $\int \sqrt{g}R$ [17]. The eigenvalues are real in the vicinity of two dimensions, and in the large-D limit, where the fixed point scaling is dominated by the $\int \sqrt{g}$ invariant [15]. θ' and $|\theta|$ are increasing functions of the dimension, for all $D \geq 4$ [17]. For the dimensions shown here, θ'' equally increases with dimension.

UV-IR connection.— A non-trivial ultraviolet fixed point is physically feasible only if it is connected to the perturbative infrared domain by well-defined, finite renormalisation group trajectories. Elsewise, it would be impossible to connect the known low energy physics of gravity with the putative high energy fixed point. A necessary condition is

θ''								
r_{mexp}	3.14	5.37	7.46	9.46	11.4	13.2	14.9	16.6
r_{exp}	3.13	5.33	7.37	9.32	11.2	13.0	14.7	16.4
r_{mod}	3.10	5.27	7.31	9.26	11.1	13.0	14.7	16.4
r_{opt}	3.04	5.15	7.14	9.05	10.9	12.7	14.5	16.2
n	0	1	2	3	4	5	6	7

Table 3: Scaling exponent θ'' in $D = 4 + n$ dimensions for various momentum cutoffs (see text).

$\lambda_* < \lambda_{\text{bound}}$, which is fulfilled. Moreover, we have confirmed by numerical integration of the flow that the fixed points are connected to the perturbative infrared domain by well-defined trajectories.

Anomalous dimension.— The non-trivial fixed point implies a non-perturbatively large anomalous dimension for the gravitational field, due to (4), which takes negative integer values $\eta = 2 - D$ at the fixed point.[1] The dressed graviton propagator $\mathcal{G}(p)$, neglecting the tensorial structure, is obtained from evaluating $1/(Z_{N,k}\, p^2)$ for momenta $k^2 = p^2$. Then the graviton propagator scales as

$$\mathcal{G}(p) \sim 1/p^{2(1-\eta/2)} \, , \tag{6}$$

which reads $\sim 1/(p^2)^{D/2}$ in the deep ultraviolet and should be contrasted with the $1/p^2$ behaviour in the perturbative regime. The anomalous scaling in the deep ultraviolet implements a substantial suppression of the graviton propagator. We verified the crossover behaviour of the anomalous dimension from perturbative scaling in the infrared to ultraviolet scaling by numerical integration of the flow (4). More generally, higher order vertex functions should equally display scaling characterised by universal anomalous dimensions in the deep ultraviolet. This is due to the fact that a fixed point action Γ_* is free of dimensionful parameters.

Cutoff independence.— Fixed points are found independently of the momentum cutoff, *e.g.* Fig. 1. The scaling exponents θ, however, depend spuriously on R_k due to the truncation. This dependence strictly vanishes for the full, untruncated flow. For best quantitative estimates of scaling exponents we resort to an optimisation, following [24, 25, 26], and use optimised values for b, for each class of cutoffs given above. Optimised flows have best stability properties and lead to results closer to the physical theory [25]. In Tab. $1-4$, we show our results for τ, θ', θ'' and $|\theta|$. The variation in θ', θ'', $|\theta|$ and τ is of the order of 11%, 5%, 7% and 4%, respectively (see Fig. 2), and significantly smaller than the variation with b [26]. With increasing n, the variation slightly increases for θ' and $|\theta|$, and decreases for θ'' and τ. The different dependences on the cutoff function, Fig. 2a,c) vs. Fig. 2b,d),

---

[1] Integer anomalous dimensions are known from other gauge theories at a fixed point away from their canonical dimension, *e.g.* abelian Higgs [31] below or Yang-Mills [32] above four dimensions.

$\lvert\theta\rvert$								
r_{mexp}	3.49	6.06	8.76	11.6	14.6	17.7	21.1	24.6
r_{exp}	3.49	6.03	8.69	11.5	14.4	17.4	20.7	24.0
r_{mod}	3.44	5.95	8.58	11.3	14.2	17.3	20.5	23.8
r_{opt}	3.38	5.81	8.35	11.0	13.8	16.8	19.8	23.1
n	0	1	2	3	4	5	6	7

Table 4: Scaling exponent $\lvert\theta\rvert$ in $D = 4 + n$ dimensions for various momentum cutoffs (see text).

indicate that the observables are only weakly cross-correlated. The expected error due to the truncation (2) is larger than the variation in Fig. 2. In this light, our results in the Einstein-Hilbert truncation are cutoff independent.

Convergence.— The convergence of the results is assessed by comparing different orders in the expansion. The fixed point persists in the truncation where the cosmological constant is set to zero, $\lambda = \beta_\lambda = 0$. Then, $\beta_g(g_*, \lambda = 0) = 0$ implies fixed points $g_* > 0$ for all dimensions and cutoffs studied. The scaling exponent $\theta = -\partial\beta_g/\partial g\vert_*$ at g_* is real and of the order of $\lvert\theta\rvert$ given in Tab. 4. The analysis can be extended beyond (2), *e.g.* including $\int \sqrt{g}R^2$ invariants and similar. In the four-dimensional case, R^2 interactions lead to a mild modification of the fixed point and the scaling exponents [11, 12]. It is conceivable that the underlying expansion is well-behaved also in higher dimensions.

In summary, we have found a unique and non-trivial ultraviolet fixed point with all the right properties for quantum gravity in more than four dimensions. Moreover, its universal characteristics are stable for the truncations and cutoffs considered. Together with the renormalisation group trajectories which connect the fixed point with the perturbative infrared domain, our results provide a viable realisation of the asymptotic safety scenario. If the fixed point persists in extended truncations, quantum gravity can well be formulated as a fundamental theory in the metric degrees of freedom.

As a first application, we discuss implications of the gravitational fixed point for phenomenological models with compact extra dimensions, where gravity propagates in a $D = 4 + n$ dimensional bulk. Under the assumption that standard model particles do not spoil the fixed point, we can neglect their presence for the following considerations. Without loss of generality, we consider n extra spatial dimensions with compactification radius L. The four-dimensional Planck scale M_{Pl} is related to the D-dimensional (fundamental) Planck mass M_D and the radial length L by the relation $M_{\mathrm{Pl}}^2 \sim M_D^2(M_D L)^n$, where L^n is a measure for the extra-dimensional volume. A low fundamental Planck scale $M_D \ll M_{\mathrm{Pl}}$ therefore requires the scale separation

$$1/L \ll M_D \,, \tag{7}$$

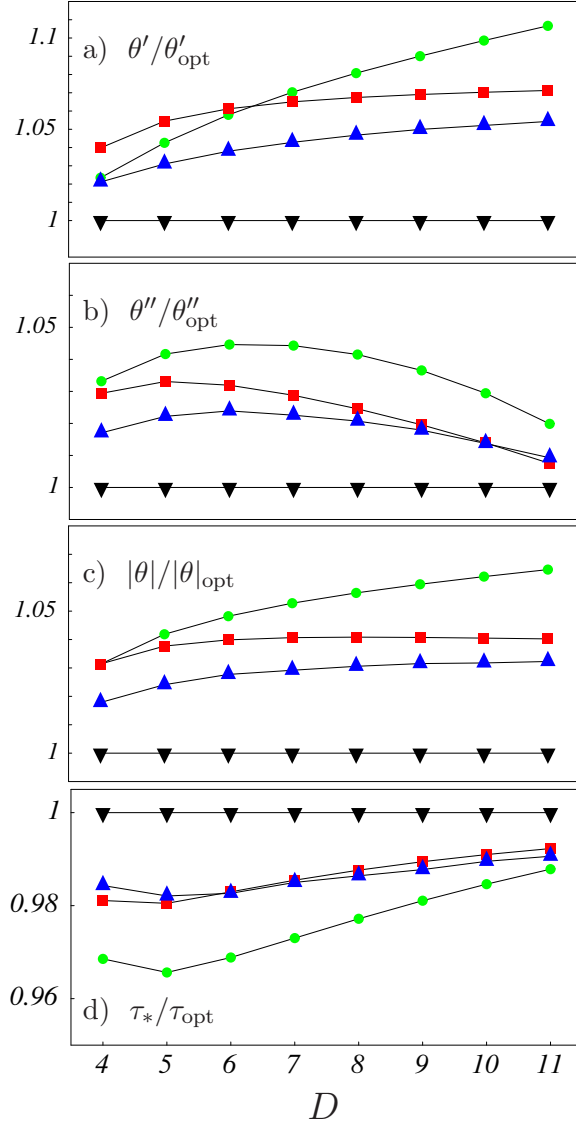which states that the radius for the extra dimensions has to be much larger than the

Figure 2: Comparison of θ', θ'', $|\theta|$ and τ for different momentum cutoffs, normalised to the result for $r_{\rm opt}$ ($r_{\rm mexp}$ ●, $r_{\rm exp}$ ■, $r_{\rm mod}$ ▲, $r_{\rm opt}$ ▼). The relative variation, for all dimensions and all observables, is very small.

fundamental Planck length $1/M_D$. For momentum scales $k \ll 1/L$, where $\eta \approx 0$, the hierarchy (7) implies that the running couplings scale according to their four-dimensional canonical dimensions, with $G_k \approx {\rm const}$. At $k \approx 1/L$, the size of the extra dimensions is resolved and, with increasing k, the couplings display a dimensional crossover from four-dimensional to D-dimensional scaling. Still, (7) implies that the graviton anomalous dimension stays small and gravitational interactions remain perturbative. This dimensional crossover is insensitive to the fixed point in the deep ultraviolet. In the vicinity of $k \approx M_D$, however, the graviton anomalous dimension displays a classical-to-quantum crossover from the gaussian fixed point $\eta \approx 0$ to non-perturbative scaling in the ultraviolet $\eta \approx 2 - D$. This

9

crossover takes place in the full D-dimensional theory. In the transition regime, following (6), the propagation of gravitons is increasingly suppressed, and the running gravitational coupling G_k becomes very small and approaches $g_* k^{2-D}$ with increasing k, as follows from (3) and (4). Therefore, the onset of the fixed point scaling cuts off gravity-mediated processes with characteristic momenta at and above M_D, and provides *dynamically* for an effective momentum cutoff of the order of M_D. For momentum scales $\gg M_D$, gravity is fully dominated by the non-perturbative fixed point and the associated scaling behaviour for vertex functions. Hence, the main new effects due to the fixed point set in at scales about M_D. The significant weakening of G_k and the dynamical suppression of gravitons can be seen as signatures of the fixed point. This behaviour affects the coupling of gravity to matter and could therefore be detectable in experimental setups sensitive to the TeV energy range, *e.g.* in hadron colliders, provided that the fundamental scale of gravity is as low as the electroweak scale. It will be interesting to identify physical observables most sensitive to the above picture.

---

[1] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B **429** (1998) 263 [hep-ph/9803315]; Phys. Rev. D **59** (1999) 086004 [hep-ph/9807344].

[2] I. Antoniadis, Phys. Lett. B **246** (1990) 377.

[3] L. Randall and R. Sundrum, Phys. Rev. Lett. **83** (1999) 3370 [hep-ph/9905221]; Phys. Rev. Lett. **83** (1999) 4690 [hep-th/9906064].

[4] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B **436** (1998) 257 [hep-ph/9804398].

[5] G. F. Giudice, R. Rattazzi and J. D. Wells, Nucl. Phys. B **544** (1999) 3 [hep-ph/9811291].

[6] S. Weinberg, in *General Relativity: An Einstein centenary survey*, Eds. S.W. Hawking and W. Israel, Cambridge University Press (1979), p. 790.

[7] R. Gastmans, R. Kallosh and C. Truffin, Nucl. Phys. B **133** (1978) 417; S. M. Christensen and M. J. Duff, Phys. Lett. B **79** (1978) 213.

[8] T. Aida and Y. Kitazawa, Nucl. Phys. B **491** (1997) 427 [hep-th/9609077].

[9] M. Reuter, Phys. Rev. D **57** (1998) 971 [hep-th/9605030].

[10] W. Souma, Prog. Theor. Phys. **102** (1999) 181 [hep-th/9907027]; gr-qc/0006008.

[11] O. Lauscher and M. Reuter, Phys. Rev. D **65** (2002) 025013 [hep-th/0108040].

[12] O. Lauscher and M. Reuter, Class. Quant. Grav. **19** (2002) 483 [hep-th/0110021]; Phys. Rev. D **66** (2002) 025026 [hep-th/0205062].

[13] M. Reuter and F. Saueressig, Phys. Rev. D **65** (2002) 065016 [hep-th/0110054].

[14] R. Percacci and D. Perini, Phys. Rev. D **67** (2003) 081503 [hep-th/0207033]; Phys. Rev. D **68** (2003) 044018 [hep-th/0304222]; R. Percacci, hep-th/0511177.

[15] D. F. Litim, Phys. Rev. Lett. **92** (2004) 201301 [hep-th/0312114].

[16] A. Bonanno and M. Reuter, JHEP **0502** (2005) 035 [hep-th/0410191].

[17] D. F. Litim, talks at RG2005, Helsinki (Aug 2005), and at XXVIII Spanish Relativity Meeting, A Century of Relativity Physics, Oviedo (Sept 2005), hep-th/0606044.

[18] P. Forgacs and M. Niedermaier, hep-th/0207028; M. Niedermaier, JHEP **0212** (2002) 066 [hep-th/0207143],Nucl. Phys. B **673** (2003) 131 [hep-th/0304117].

[19] H. W. Hamber, Phys. Rev. D **61** (2000) 124008 [hep-th/9912246]; Phys. Rev. D **45** (1992) 507; H. W. Hamber and R. M. Williams, Phys. Rev. D **70** (2004) 124007 [hep-th/0407039]; H. W. Hamber and R. M. Williams, hep-th/0512003.

[20] J. Ambjorn, J. Jurkiewicz and R. Loll, Phys. Rev. Lett. **93**, 131301 (2004) [hep-th/0404156]; Phys. Rev. Lett. **95** (2005) 171301 [hep-th/0505113].

[21] V. Branchina, K. A. Meissner and G. Veneziano, Phys. Lett. B **574** (2003) 319 [hep-th/0309234]; J. M. Pawlowski, hep-th/0310018.

[22] J. Berges, N. Tetradis and C. Wetterich, Phys. Rept. **363** (2002) 223 [hep-ph/0005122]; J. Polonyi, Centr. Eur. Sci. J. Phys. 1 (2002) 1 [hep-th/0110026].

[23] D. F. Litim and J. M. Pawlowski, in *The Exact Renormalization Group*, Eds. Krasnitz et al, World Sci (1999) 168 [hep-th/9901063].

[24] D. F. Litim, Phys. Lett. **B486** (2000) 92 [hep-th/0005245]; Phys. Rev. D **64** (2001) 105007 [hep-th/0103195]: Int. J. Mod. Phys. A **16** (2001) 2081 [hep-th/0104221].

[25] D. F. Litim, Nucl. Phys. B **631** (2002) 128 [hep-th/0203006]; JHEP **0111** (2001) 059 [hep-th/0111159]; Acta Phys. Slov.**52** (2002) 635 [hep-th/0208117]; JHEP **0507** (2005) 005 [hep-th/0503096].

[26] P. Fischer, D. F. Litim, talk at Einstein's Century Meeting, Paris (July 2005), hep-th/0606135.

[27] J. M. Pawlowski, D. F. Litim, S. Nedelko and L. von Smekal, Phys. Rev. Lett. **93** (2004) 152002 [hep-th/0312324]; J. M. Pawlowski, hep-th/0512261.

[28] M. Reuter and C. Wetterich, Phys. Rev. D **56** (1997) 7893 [hep-th/9708051]; D. F. Litim and J. M. Pawlowski, Phys. Lett. B **435** (1998) 181; JHEP **0209** (2002) 049 [hep-th/0203005]; F. Freire, D. F. Litim and J. M. Pawlowski, Phys. Lett. B **495** (2000) 256 [hep-th/0009110]; J. M. Pawlowski, Int. J. Mod. Phys. A **16** (2001) 2105, Acta Phys. Slov. **52** (2002) 475.

[29] D. F. Litim, J. M. Pawlowski and L. Vergara, hep-th/0602140.

[30] H. Kawai, Y. Kitazawa and M. Ninomiya, Nucl. Phys. B **393** (1993) 280 [hep-th/9206081].

[31] B. Bergerhoff, F. Freire, D. F. Litim, S. Lola and C. Wetterich, Phys. Rev. B **53** (1996) 5734 [hep-ph/9503334]; B. Bergerhoff, D. F. Litim, S. Lola and C. Wetterich, Int. J. Mod. Phys. A **11** (1996) 4273 [cond-mat/9502039]; I. F. Herbut and Z. Tesanovic, Phys. Rev. Lett. **76** (1996) 4588 [cond-mat/9605185].

[32] D. I. Kazakov, JHEP **0303** (2003) 020 [hep-th/0209100]; H. Gies, Phys. Rev. D **68** (2003) 085015 [hep-th/0305208]; T. R. Morris, JHEP **0501** (2005) 002 [hep-ph/0410142].